Choice of the Hough transform for image registration

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ABSTRACT

Image registration algorithms should be robust against partly erroneous and inconsistent data. The evidence accumulation mechanism known as the Hough Transform (HT) finds the solution indicated by the largest consistent subset of the data. The important case of feature-based registration under the simplified affine transformation, that is, translation, rotation and isotropic scaling, can be easily stated in the terms of HT. Until recently, the use of HT in the considered application was prohibited by excessive computational requirements, but the development of the hardware permanently relieves these limitations. Three versions of the HT, both in the crisp and fuzzy version, were examined against the test images: the Generalised HT (GHT), the Modified Iterated HT (MIHT), and the version called here the Direct Accumulation HT (DAHT), known also as GIPSC on the grounds of photogrammetry. The results indicate that the fuzzy DAHT is robust for over 50% of errors in data, fuzzy GHT up to nearly 30%, and that all the crisp versions as well as the fuzzy MIHT are fragile at least for some examples. The practical applicability of the DAHT and GHT is shown for hierarchical registration of simulation and portal images used in quality assessment of oncological radiotherapy.

Keywords: image registration, feature-based, Hough transform, evidence accumulation, robustness, outlier elimination.

1 INTRODUCTION

1.1 The problem of image registration

Joint analysis, or at least joint visualisation of images showing the same object, but coming from different imaging modalities, makes it possible to derive more information than in the case of the separate analysis of these images. The process of bringing the images to a common coordinate system is called the *registration* process*. The problem of image registration resolves itself to finding the transformation of one of the images into the other one. We shall call the transformed image the *overlaid image*, and other image – the *reference image*. The transformation can pertain to the geometry (image coordinates) as well as to the contents (brightness or colour) of the registered images.

The criterion according to which the registration is carried out depends on its purpose, but generally it expresses what should be understood as "good registration" in the given case. Let us take into account two images, each showing a human face, not necessarily the face of the same person. As the "good registration" of these images we can consider for example such one, which leads to overlaying the centres of the pupils. Another example is to overlay all the edges present in one image onto the corresponding edges in the other one. The third criterion of a "good registration" can require that all the points which correspond to each other in both images were overlaid. The requirement of the first example is easy to fulfil in general: two points of one image should be overlaid onto two points in the second one, the points are well defined and in most cases there are no doubts which point should be overlaid onto which. Such points will be called the *corresponding points*. The pupils from the first example and the edges from the second one will be called the *features*, calculated from the raw data, that is, from the images themselves. The following question should be put forward in the beginning: which point of the feature in the overlaid image corresponds to a given point of the feature in the reference image? This information is going to be known *after* the registration is performed, while actually it would be necessary *before* this process is started. It may well happen that some points of one image have no counterparts in the other one: the images can be made in different light, different fragments of the faces can be visible, and finally, the faces can be significantly different. In the case of the third

* Other denotations used are: *image coregistration*, *image matching* and *image alignment*.

above given criterion all the image data should be considered, so the problem is even more difficult. It is not even known, if the bright points of one image correspond to bright, or to dark points of the other one. In the first and second example, the feature-based registration is described, while in the third example the registration based on raw data is presented. In the first two examples the transformation of the image brightness or colour is not relevant, while in the third one such transformation should also be considered.

From this illustrative example it follows that the image registration problem can be formulated in various ways. Several systems of classification of the registration problems can be found in the literature[1, 2, 3, 4]. These systems can be brought to ten criteria[5, 6]. We shall not list all of them here; let us only indicate that the methods having the following features will be interesting to us in this paper: 1º feature-based; 2º linear; 3º interpolating and 4º fully automatic. In the *interpolating* methods, it is attempted to *accurately* overlay the *selected* features, in opposition to the *approximating* ones, where the target is to minimise the *error measure* in the *whole* set of the features given. This means that the method sought should *automatically* select the *corresponding features* from those given and register them accurately, while leaving the non-corresponding features unregistered. As an example of the registration we seek, the Fig. 2 a1-c1 can be shown. In this figure the black pixels can be considered as the feature pixels. We have therefore the case of registering the binary images.

1.2 State-of-the-art and the missing knowledge

The extensive presentations and discussion of the state-of-the-art in the domain of image registration can be found in the above cited papers[1, 2, 3, 4] and in the article by Kozińska and Chmielewski[5]. The important conclusion which can be drawn from these presentations is that there are at least two vital problems pertaining to this domain. The first of them is the problem of correspondence, already outlined in the previous chapter. The second one is the problem of the dimensions of the set of data which need to be analysed. The dimensions of the images can be large. Also the dimensions of the space of the sought parameters of the transformation can be large. If the registration method is expected to be fully automatic, then this space, called the *parameter space*, should be analysed in its entirety. The known method which have the necessary potential for the efficient analysis of this parameter space, in the presence of the inconsistencies in the data, is the Hough transform (HT)[7], together with the large group of related methods derived from it. The HT can be considered as the *evidence accumulation* method (see the papers by Maitre[8], Illingworth and Kittler[9] and Leavers[10] and the book by Leavers[11] for reviews). Within the HT-based methods, the Generalised Hough Transform (GHT)[12] seems to be a perfect tool for the considered task. Especially, its ability of properly coping with data containing errors and having gaps, is of special importance. The GHT in its original form aims at detecting the presence of an arbitrary pattern in the image. The pattern is given by an example, which can be represented in the form of another image. Both images are binary; hence, we have the same case as that of the registration of binary images containing the features, or in other words, the feature-based image registration. However, to the best knowledge of the author of the following paper, in the application to image registration this method has been very rarely encountered until quite recently. The only publications earlier than 1999 found in extensive searches of the available internet data bases were the papers by Costabile and Pieroni[13] and Cideciyan and Nagel[14]. In the first of these papers[13], the use of the GHT was limited to images containing the polygons. In the second one[14], the plain HT was used only to detect the objects to be overlaid, while the registration was made with no relation to HT. Strange to say, the GHT in its first version seems not to have been used for image registration of arbitrary images until now.

In 1999, Habib and Schenk[15] proposed the Modified Iterative Hough transform (MIHT) for image registration under perspective projection and for other applications. MIHT was further developed in the subsequent papers[16, 17, 18]. Recently, Seedahmed and Martucci[19], and independently – but evidently later – Chmielewski[20], proposed the use of another version of the HT for image registration under the simplified affine transformation, consisting of the translation, rotation and uniform scaling. Seedahmed and Martucci called the proposed method the Geometrically Invariant Parameter Space Clustering (GIPSC), while Chmielewski used the notion of Direct Accumulation of Parameters, giving the rise to the name Direct Accumulation HT (DAHT). In fact, the versions of HT equivalent to GIPSC or DAHT have been described earlier in the application to the problems other than image registration, for example, to the detection and description of ellipses in an image[11]. Much earlier – in 1985 – Maitre[8] introduced the classification of the Hough transforms into two classes: 1 to *m* and *m* to 1 HT. In the sense of that classification, the MIHT and DAHT are the limit cases of the *m* to 1 HT in the sense that the sufficiently large set of image features (*m*) gives rise not only to one (1) curve in the parameter space, but to just one point. MIHT goes further, in that the parameter space is decomposed into the set of lower-dimensional subspaces, and in all of them the sufficiently large sets of features give rise to single points. To make this decomposition possible in the general case, it is assumed that the convergence of this iterative method is not disturbed by the fact that the preliminary estimates of the yet unknown parameters are used as the initial condition.

In the above cited papers on MIHT and GIPSC, examples of the results for the test pairs of images were given, and the statements on the robustness of the methods against noise in the images were made. The real-life test images were used, so the amount of noise or features having no correspondence resulted from the nature of these images and could not be controlled. What seems to be missing is a more systematic analysis of the robustness of the compared methods, including the hitherto forgotten GHT. In this paper such an analysis is undertaken. Two pairs of artificial test images with the regulated amount of noise are registered. We remain on the grounds of experiments with test images, so an argument of the lack of a theoretical analysis can be raised. Nevertheless, having in mind that in complex cases the experiment frequently precedes the theory, and further, that even one negative example disproves the theorem, we believe that the results presented here can shed some light on the problem of choice of the right method for feature-based, automatic image registration.

In the following chapters, the GHT, DAHT and MIHT methods will be briefly described. Their computational complexity will be compared. Further, the results of functional tests of the methods with two pairs of artificial images and a pair of real-life ones coming from a biomedical application will be presented and discussed. The conclusions will close the paper.

2 METHODS

2.1 The Generalised Hough Transform

The Generalised Hough Transform (GHT) will be presented according to the detailed description of the paper by Ballard[12] found in the survey by Illingwoth and Kittler[9]. The method is applied to detect a pattern which can appear in the investigated binary image in a position defined by a translation by a vector $[T_x, T_y]$, rotation by an angle α and scaling with a scale s, with respect to the initial position of the pattern given as a template in the form of another binary image. It should be noted that the method imposes no requirements on the shape of the template. The centre of rotation and scaling c_x, c_y is arbitrary and constant. In the application of our interest we treat the pattern as the overlaid image O, and the investigated image as the reference image R. They have a common coordinate system. Hence, we have the problem of image registration with the linear transformation having four parameters. Let us consider a pair of selected pixels: x_O, y_O in the overlaid image and x_R, y_R in the reference image. If these pixels are corresponding, then

$$\begin{aligned} T_x &= x_R - x_c + s(x_c - x_O)\cos\alpha - s(y_c - y_O)\sin\alpha\,, \\ T_y &= y_R - y_c + s(x_c - x_O)\sin\alpha + s(y_c - y_O)\cos\alpha\,. \end{aligned} \tag{1}$$

Let us divide the range of variability of each of the parameters into n_x, n_y, n_s, n_α intervals, respectively. For each parameter its intervals have uniform length. Then, the parameters having real values can be indexed with the indexes of the intervals, which we shall denote as i_x, i_y, i_s, i_α , with the accuracy equal to the lengths of the intervals. Let us form a four-dimensional array of integers A, called the *accumulator array*, indexed with these indexes. The values of the elements, initially zero, will be incremented in the following way. For *each pair* of pixels – one feature pixel of the overlaid image, one feature pixel of the reference one – it is possible to plot the hypersurface described by the equations (1) in the parameter space. The number of these pairs is $n_O * n_R$, where n_O is the number of feature pixels in the overlaid image, and n_R – that in the reference image. The elements of the accumulator array crossed by the hypersurface are incremented by 1. The simplest way to perform this is as follows. For all the possible pairs i_s, i_α find s and α, calculate T_x, T_y, find their indexes i_x, i_y, and increment $A[i_x, i_y, i_s, i_\alpha]$. After the process is finished, the indexes of the maximum element of A indicates such parameters of the transformation, which correspond to the consistent transformation of the largest set of feature pixels from the image O into the image R. This is the sought registering transformation of these images, found with the accuracy corresponding to the lengths of the intervals represented by the indexes of the parameters.

The question which arises is how long the intervals should be. In the present paper, we have used the "reasonable" values, corresponding to the expected accuracy of the methods: 0.5 pixel for translation, 0.5 deg for angle, and 0.01 for scale. These values were used for all the compared methods in all the examples presented further, except those for a real-life medical images, where variable lengths were used in a hierarchical manner.

The accumulation process can well be compared to the process of voting, with the simple majority vote, for a subset of the set of possible quantised values of the parameters. Each increment made in the accumulator corresponds to one vote. The subset – the quadruple of indexes – which is the winner of the voting process is that which has the largest set of related image feature data, yielding consistent votes for the parameters. Hence, it can be expected that the result of the method should be correct as long as not less than the half of the data are correct.

2.2 The Direct Accumulation Hough Transform

The approach of the Geometrically Invariant Parameter Space Clustering (GIPSC)[19], equivalent to Direct Accumulation HT (DAHT)[6], is somewhat different. For a univocal assessment of the four parameters of the transformation it is necessary to have two corresponding pixels in each of the registered images, that is, one line segment in each image. Two pairs of pixels give rise to four equations. Without knowing the correspondence, let us take *all the possible pairs – all the possible combinations* of segments, and for each pair of segments let us solve the system of four equations like (1). This is not difficult, as the scale s and the angle α between the segments is easy to calculate. It is necessary to consider two angles differing by 180^o, as it is possible to register a segment with another segment in two ways. Then, the translations are found directly from (1). After calculating the four parameters for the given pair of segments, the respective accumulator element is incremented[**]. As it can be seen, the method in its basic formulation is extremely simple. Once more, the process of making a single increment for a given pair of segments can be considered as making one vote for the resulting parameters.

It can be mentioned marginally, that in addition to the accumulation in 4D, also the separate accumulation in four 1-dimensional accumulators for four parameters was attempted. This was equivalent to analysing the projections instead of the full 4D accumulator array, to save memory. Some promising results were received for scale, but generally the results were unstable from example to example. At the present stage the attempt was considered as a failure and will not be presented here.

2.3 The Modified Iterated Hough Transform

Using the DAHT as the starting point, we shall describe the MIHT[16] adapted to our problem of interest as its extension. The DAHT have two disadvantages. The first is the necessity of analysing all the possible combinations of pairs of pixels in the overlaid and in the reference image, which gives rise to very large numbers of operations for larger images (see next chapter). The second is the volume of the accumulator array, which is 4-dimensional. Both these disadvantages are relieved by introducing the following iterative scheme. Let us assume for a while that form the four parameters sought: T_x, T_y, α and s, only the angle α is unknown, and the remaining three are given. Then, α can easily be calculated form the system (1), using only the coordinates of one pair of pixels: one in the reference image and one in the overlaid image. To this end, $\sin \alpha$ and $\cos \alpha$ are calculated as the independent unknowns of the set of linear equations, and α is calculated from them. We shall not cite the simplistic equations here. The number of combinations is much smaller now (pixels instead of pairs of pixels), and the dimensionality of the accumulator array for the single parameter α is simply one. Now, removing the assumption of three known parameters, we can take their estimates as initial values and perform the accumulation for α as just described, for all the existing pairs of feature pixels in the registered images. With the resulting value of α the same procedure can be repeated for the scale s, retaining the estimated values for only the two remaining parameters. It can be noted that each of the equations (1) gives rise to one value of s, so both are accumulated. Finally, the translations are simultaneously accumulated in a 2D accumulator, using directly the equations (1). Starting with the received results, the process is repeated iteratively until the results stabilise. In the present work, the stabilisation is meant as that none of the differences between the indexes of parameters in the accumulator arrays after the present and after the previous iteration is larger than one.

Three problems arise with MIHT. First, which values of the parameters should be taken as the initial ones. Second, in which sequence the parameters should be estimated. Third, whether the process is convergent. In the available papers on MIHT the question of the sequence of parameters is treated for the case of registration under different transformation than the one of our interest, and the question of convergence remains open. So it remains in this paper. The trials with all the six possible sequences of parameters were performed, with very similar results. The sequence finally used in the presented results is: (translation, scale, angle) – that one which led to the most accurate results for the example of real-life medical images, presented further. As the initial values, the only generally reasonable, neutral ones were used: those who correspond to null transformation, that is, translation and rotation angle zero, and scale one. In the testing of the software developed for this paper, the known accurate values of the parameters were used for all the examples, and as expected, the MIHT stopped with the same values as results after one iteration of accumulations.

[**] As described by Seedahmed and Martucci[19], GIPSC includes the second stage, in which the corresponding features revealed by the HT approach are used to improve the registration accuracy with the least squares method. This stage is not related to HT and as such it is not considered here.

2.4 Computational complexity of the compared methods

It is easy to note that in the classical GHT the formulae (1) are calculated C_{GHT} times, where

$$C_{GHT} = n_O * n_R * n_s * n_\alpha \ . \tag{2}$$

If $n_O \sim n_R$ and $n_s \sim n_\alpha$, then the computational complexity will be of the order $O(n^2)$ with respect to the number of feature points in the images, and also $O(n^2)$ with respect to the resolution of the accumulator, as far as the parameters s and α are considered. The volume of the accumulator is

$$V_{GHT} = n_x * n_y * n_s * n_\alpha \ . \tag{3}$$

In DAHT, the formula for the number of calculations for the pairs of intervals is

$$C_{DAHT} = r_O * n_O * (n_O - 1) * r_R * n_R * (n_R - 1) / 4 \ . \tag{4}$$

It results from the calculation of the number of segments which can be formed from a given number of points as ends, and the coefficients r_O and r_R denote the share of the segments taken into account in the calculations, due to their sufficient length, with respect to all the segments. If $n_O \sim n_R$, then the computational complexity will be of the order $O(n^4)$. the volume of the accumulator is the same as in GHT:

$$V_{DAHT} = V_{GHT} = n_x * n_y * n_s * n_\alpha \ . \tag{5}$$

In the case of MIHT, the number of calculations depends not only on the dimensions of the problem, but also on the number of iterations n_i until the solution stabilises. In the convergent cases this number is small: 3 to 20.

$$C_{MIHT} = 3 * n_O * n_R * n_i \ . \tag{6}$$

If $n_O \sim n_R$, then the computational complexity will be of the order $O(n^2)$ with respect to the number of feature points in the images, as in GHT, but without relation to the resolution of the accumulator. The total volume of the accumulators is

$$V_{MIHT} = n_x + n_y + n_s + n_\alpha \ . \tag{7}$$

The MIHT is definitely the fastest and least memory-consuming method from the three considered ones.

Even for small problems the above formulae for the complexity of GHT and especially DAHT yield large results, reaching up to hundreds of millions and more (see Tab. 1, 3 and 4 further). It can be supposed that this is the reason why the methods have arisen a small interest until now. The same tables indicate however, that given the efficiency of even the simple computational equipment, the considered methods are within the scope of usability, for certain ranges of application. Given the constant development of the processors it can be expected that this range will broaden. The good moment for paying attention to the methods using the Hough transform and the related concepts to image registration is now.

2.5 Constraints reducing the complexity of the direct accumulation

In the GHT method the complexity decreases with the number of intervals into which the ranges of parameters are divided, that is, with the resolution of the accumulator. Hence, a good initial estimation of these ranges directly shortens the calculations. In the DAHT method, which is the most numerically requiring, larger savings can be received by reducing the number of feature points, that is, the resolution of the image. The resolution of the accumulator is not important. If the image in the reduced scale is calculated by setting a feature in its pixel if at least one of the pixels of the original image falling inside this pixel is a feature pixel, then the amount of missed information is relatively small (Fig. 5 b, c). Although in DAHT the reduction of the ranges of variability of the parameters discards a significant number of considered pairs of the pixels, as they do not meet the condition of the scale or angle being within the admissible range, but these constraints necessitate for performing the relevant tests on each pair, which takes time. It is possible to consider the reduction of the search space with the criterion based on an ordering of pairs, similarly to what was previously proposed in a different application[21]. As the ordering parameter, the angle or the length of the segments can be used, or both of these parameters in a hierarchical way. Having a sequence of pairs, it is necessary to find only the first and the last pair fulfilling the given condition. In the present version of the method only the condition of sufficient length of the considered interval is used, which can be considered as a simplification. Discarding the segments shorter than a given length, for example, half of the largest dimension of the objects present in the images, gives significant time savings.

In conclusion it can be stated that for the GHT as well as for the DAHT there exist the ranges of parameters of the problems, for which these methods are applicable and competitive. For example, the DAHT can be used for preliminary estimation of the transformation parameters for the images in a reduced resolution, irrespectively of the resolution of the accumulator array. After the estimation is made, the GHT or another method can be applied, for example, the iterative method requiring a reasonable starting point[22, 23], for the images in the full resolution. Needless to say, the MIHT is always superior to the two remaining methods, as far as computational requirements are considered.

2.6 Experiment with the fuzzy accumulation

A trial of applying the fuzzy accumulation process in the compared methods was carried out. The Hough transform in the fuzzy version has been described in the most complete way by Strauss[24]. Here, a simplified version was applied. It consists in incrementing, together with the given accumulator element, also its neighbouring elements. The shape of the neighbourhood and the shape of the function describing the number by which the elements are incremented, defined on this neighbourhood, represents the uncertainty related to the values of the parameters pertaining to the particular vote in the accumulation process. In our application, a 3*3 neighbourhood was used, with the central value of 3, and the remaining values equal to 2. The fact that only the immediate neighbours of the given vote in the indexed parameter space were taken into account represents the possibility of making an error of 1 in indexing the accumulator. The mask used has the following property. If the accumulator elements which have the indexes differing by less than 2 are incremented, the maxima created by the incrementation merge together (see Fig. 1). In the MIHT, where the accumulator arrays are 1D, the 1*3 linear mask with values 2, 3, 2 was used.

```
2 2 2              2 2 2              2 2 2              2 2 2              2 4 4 2            4 4 4
2 3 2              2 3 2              2 3 4 2 2          2 5 4 2            2 5 5 2            4 6 4
2 2 2  2 2 2       2 2 4 2 2          2 2 4 3 2          2 4 5 2            2 4 4 2            4 4 4
       2 3 2           2 3 2              2 2 2            2 2 2
       2 2 2           2 2 2
     a                  b                  c                  d                  e                  f
```

Figure 1. a: masks of the fuzzy incrementation of the accumulator elements. b-f: Process of voting for the neighbouring elements differing in indexes by a decreasing difference – the maximum tends to be between the centres of the elements instead of in these centres, thus promoting the cooperative voting.

In the paper by Strauss[24] a broad discussion of the compromise between the precision and certainty of the results can be found. The fuzzy approach to accumulation is the way of making this compromise less restrictive, by separating the problems of precision and uncertainty, at least to a certain degree. In the present paper this problem is not central, so the interested reader is encouraged to refer to the source paper[24]. Here, let us note only that if the structure of the masks made the tested algorithms less sensitive to errors by one accumulator cell, then the range of expected accuracy spanned between 0.5 and one pixel for translation, 0.5 and one deg for rotation, and between 0.01 and 0.02 for scale. Such ranges seem natural for the binary images used.

3 EXAMPLES OF CALCULATIONS

3.1 Artificial test images and the robustness against errors in data

The First two tests were performed with a series of artificially generated, binary test images, 50*50 pixels large, each containing a rectangle. One of the images contained 60 feature pixels, and the other one – 120. In each image the errors were introduced in such a way that the location of *b*% randomly chosen feature pixels (black pixels) was changed into another randomly chosen location, with the uniform probability density in all the image (Fig. 2). This was done for *b* from 0 to 70, with the step 1. This makes 71 pairs. In the case of the small image having only 60 pixels, some of the images for subsequent shares of errors differed only with the locations of disturbed pixels, not by their number. The registration was made in both directions – *large onto small* as well as *small onto large*. This gave rise to two tests, denoted as "l" for *large onto small* and "s" for *small onto large*.

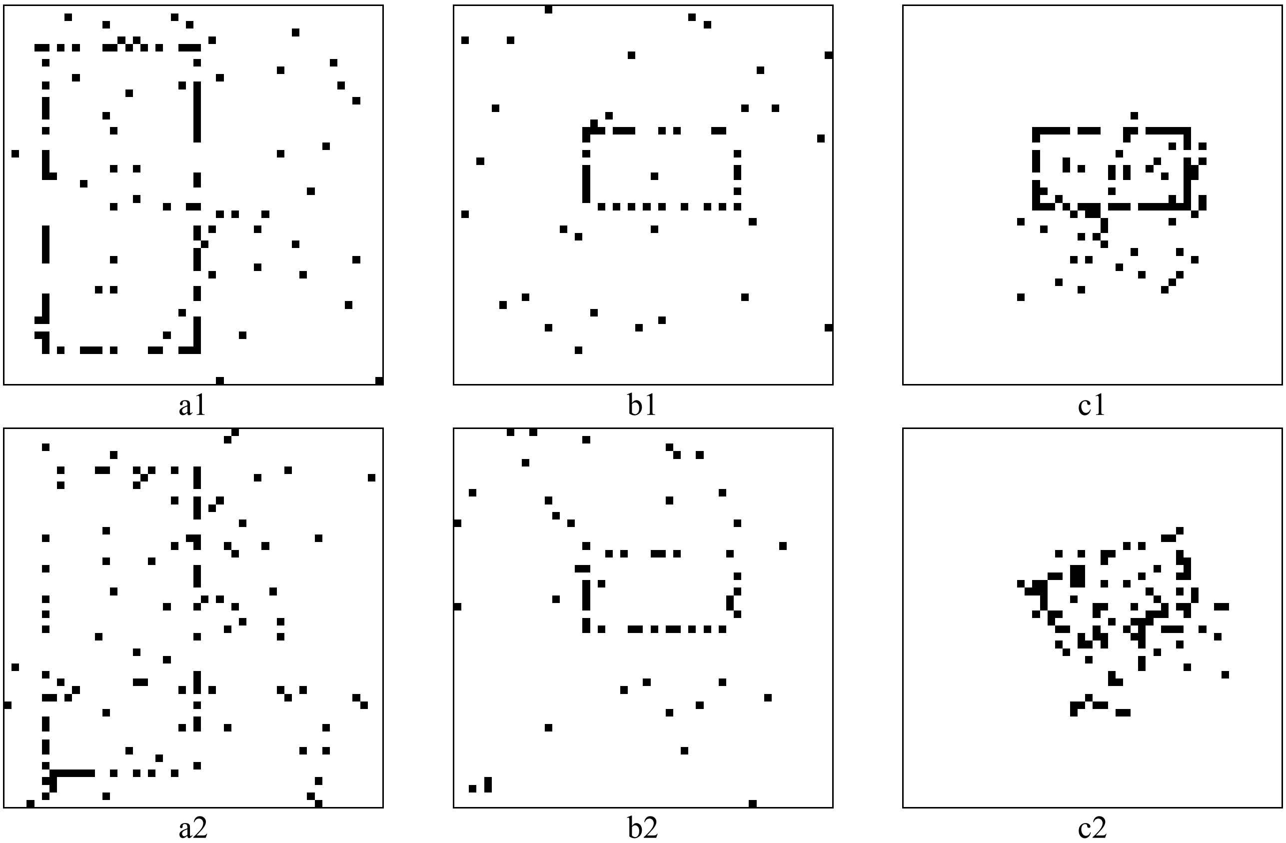


Figure 2. The test images "l" and "s" and selected results for the fuzzy DAHT algorithm. a1-c1: selected correct result found for 50% error share in data. a2-c2: selected incorrect result, for 57% error share. a: overlaid images; b: reference images; c: results: image a transformed into the coordinates of the image b.

The third test was made with a pair of images which were intended to simulate a more realistic case, in which the correspondence was not as evident as in the previous examples, and the accurate registration of each pixel was not possible. The overlaid and reference images were both derived from the phantom shown in Fig. 3a. The overlaid image was derived from this image by rotation by 5 deg to left (Fig. 3b). The reference image was derived by rotating the phantom by 10 deg to right and scaling the image up with the scale 1.2 (Fig. 3c). In this way, the pair of images was produced for which the transformation was described by α = 15 deg, s = 1.2, and the resulting $T_x \approx 11$ and $T_y \approx 23$. As previously, the series of pairs of images with noise from 1 to 70% was formed. This test was denoted "p" for "phantom".

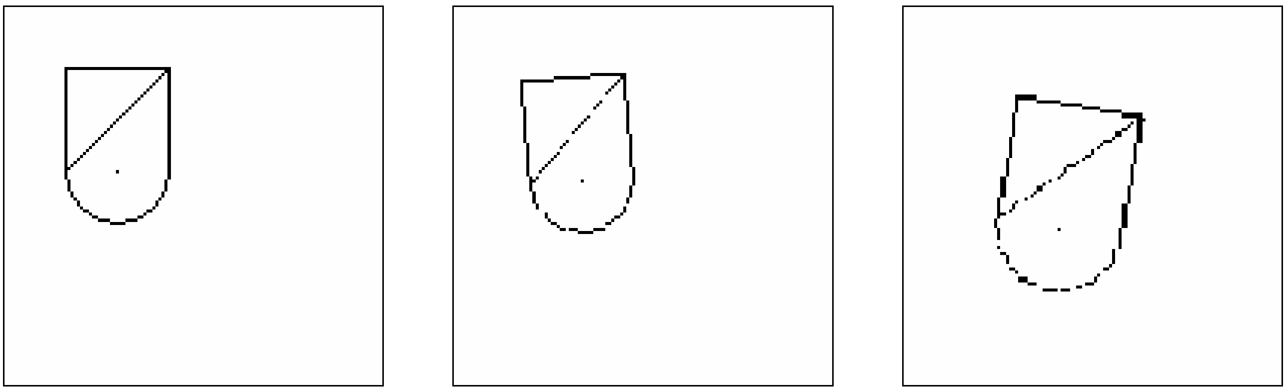
In total, there are 3 sets of tests, each having 71 pairs of images. For these three tests, results were found with the GHT, DAHT and MIHT, all of them both in the crisp and the fuzzy versions. This gave rise to 18 series of results, each consisting of 71 single results. The series were denoted with three letters each. First letter for the test – "l", "s" or "p", as described above. Second letter for the method – "g" for GHT, "d" for DAHT and "m" for MIHT. Third letter for version – "c" for crisp, "f" for fuzzy.

The ranges of the parameters, lengths and numbers of intervals in the accumulators, volumes of the accumulators and average times of the calculations were collected in the Tab. 1.

For each result in each series, the measure of the registration accuracy was calculated. This was the maximum taken from the errors of locations of the characteristic points after the registration. For the series "l" and "s" these were the vertices of the rectangles. For the series "p", these were the four vertices of the square and the lowermost pixel of the semicircle. These measures of accuracy, or in fact, error measures, were extrinsic with respect to the methods, because the knowledge on the accurate registration result was necessary to construct them.

test	parameter	T_x	T_y	α	s	GHT		DAHT		MIHT	
						V	t [s]	V	t [s]	V	t [s]
	range	-10.00÷10.00	-10.00÷10.00	45.00÷135.00	0.10÷1.10						
l	Δ; n	0.50; 41	0.50; 41	0.50; 181	0.01; 101	30.7 M	201	30.7 M	200	364	<1
	result	2.00	1.00	90.0	0.50						
	range	-10.00÷10.00	-10.00÷10.00	-135.00÷-45.00	1.50÷2.50						
s	Δ; n	0.50; 41	0.50; 41	0.50; 181	0.01; 101	30.7 M	97	30.7 M	83	364	<1
	result	-2.00	4.00	-90.00	2.00						
	range	0.00÷20.00	10.00÷30.00	-10.00÷20.00	0.80÷1.60						
p	Δ; n	0.50; 41	0.50; 41	0.50; 61	0.01; 81	8.3 M	88	8.3 M	655	224	<1
	result	11.00	23.00	15.00	1.20						

Table 1. Ranges of the parameters, lengths Δ and numbers n of intervals in the accumulators, accurate results, volumes of the accumulators V (numbers of elements) and average times of computations t for the three tests (fuzzy versions) with the generated images: “l” for *large rectangle onto small*, “s” for *small onto large*, “p” for *phantom*. M = 10^6. Time for Pentium 1000 MHz.



a b c

Figure 3. The test images “p” intended to simulate a more realistic case then the rectangles of Fig. 2.

These error measures received for the 18 series of results are shown in the graphs in Fig. 4. In each family of graphs for the given method, the graph for the test and version for which the result appeared the most robust was marked with points with a line, and the remaining graphs – only with points.

For GHT, the fuzzy versions were robust up to nearly 30% of errors in data. The robustness was not uniform for the three test examples. For DAHT (or GIPSC), all the fuzzy versions were robust up to over 50%, uniformly for the three test examples. (The results for the *phantom* of the crisp GHT and DAHT were both always erroneous.) The results for MIHT were correct only for small shares of errors in data, and only for the test examples of the rectangles. The lack of convergence appeared frequently for all the examples. It is interesting that the simple fuzzification used in the present study made such a significant change in the functionality of the considered methods.

3.2 Natural images

The DAHT method and the compared GHT method were tested on pairs of images coming from actual, clinical measurements performed in order to asses the quality of oncological radiotherapy. Quality assessment is made by comparison of the images coming from therapy planning – the *simulation images*, and the those coming from the execution of the therapy – the *portal images*[25, 26]. The features were found semi-automatically, by the manual choice and editing, if necessary, of the edges found with the *zero-second-derivative* detector (Fig. 5 a1, a2). The registration was performed fully automatically, with the considered DAHT and GHT methods. The results for one pair of images in a three-level resolution pyramid have been presented in Tab. 2. The selected results have been shown in Fig. 5 b, c. The DAHT and GHT methods, in the above described version with fuzzy accumulation, gave the same final results. There was a difference in the intermediate result at the resolution level 1:4 – the difference in angle α by 1°, which corresponded to a difference in the index i_α by one. This insignificant difference has not been shown in Tab. 4. The computational complexity and calculation times have been shown in Tabs. 3 and 4. The examples are a good illustration of the combinatorial increase of complexity in GHT for the increasing size of the accumulator array and in DAHT for the increasing resolution of the image.

The fact that the results for MIHT for this pair of images needs an explanation. In fact, the MIHT gave no accurate result for this pair at the resolution 1:4. For the initial values of the parameters considered as neutral, as described in Section 2.3, the algorithm stabilised at erroneous values of the parameters, irrespective of their sequence. For the sequence (translation, scale, angle) the errors were the smallest, but still considerable (as previously mentioned, if as the initial values the correct ones were used, the algorithm stopped at these values in one iteration). This seems to suggest that MIHT is not always convergent.

resolution	parameter	T_x	T_y	α	s
1:4	range	-50.00÷50.00	-50.00÷50.00	-90.00÷90.00	0.76÷1.26
	Δ; n	1.00; 101	1.00; 101	1.00; 181	0.02; 26
	result	**1.00**	**4.00**	**1.00**	**1.10**
	new range	-1.00÷3.00	2.00÷6.00	-1.00÷3.00	1.06÷1.14
1:2	range	-2.00÷6.00	4.00÷12.00	-1.00÷3.00	1.06÷1.14
	Δ; n	0.50; 17	0.50; 17	0.50; 9	0.01; 9
	result	**2.00**	**9.00**	**0.00**	**1.11**
	new range	1.00÷3.00	8.00÷10.00	-1.00÷1.00	1.09÷1.13
1:1	range	2.00÷6.00	16.00÷20.00	-1.00÷1.00	1.09÷1.13
	Δ; n	0.25; 17	0.25; 17	0.25; 9	0.005; 9
	result	**3.75**	**18.00**	**0.50**	**1.11**

Table 2. Collected results for natural images in subsequent resolutions. Results for DAHT and GHT are identical, except an insignificant difference in angle by 1° at resolution 1:4. Δ, n: interval width and number of intervals in the accumulator array.

resolution	pixels		min. segment length	segments f.c. / all segments		pairs f.c. / all pairs	constraints together	time [min]
	O	R		O	R			
1:4	196	224	20	12K/19K=61%	16K/25K=64%	86M/177M=49%	19%	12
1:2	385	446	40	45K/74K=60%	63K/99K=64%	4.1M/2690M=0.15%	0.058%	62
1:1	712	844	80	152K/253K=60%	226K/356K=64%	3.2M/32919M=0.01%	0.0037%	741

Table 3. Complexity and time requirements of DAHT for natural images. O: overlaid image; R: reference image; f.c.: fulfilling the constraints; time given for PC – Pentium 1000 MHz.

resolution	pixels		n_α	n_s	no. of operations	time [min]
	O	R				
1:4	196	224	181	26	202M	12
1:2	385	446	9	9	14M	0.04
1:1	712	844	9	9	48M	0.12

Table 4. Complexity and time requirements of GHT for natural images. O: overlaid image; R: reference image; n_α, n_s: numbers of intervals in the accumulator array for angle and scale; time for Pentium 1000 MHz.

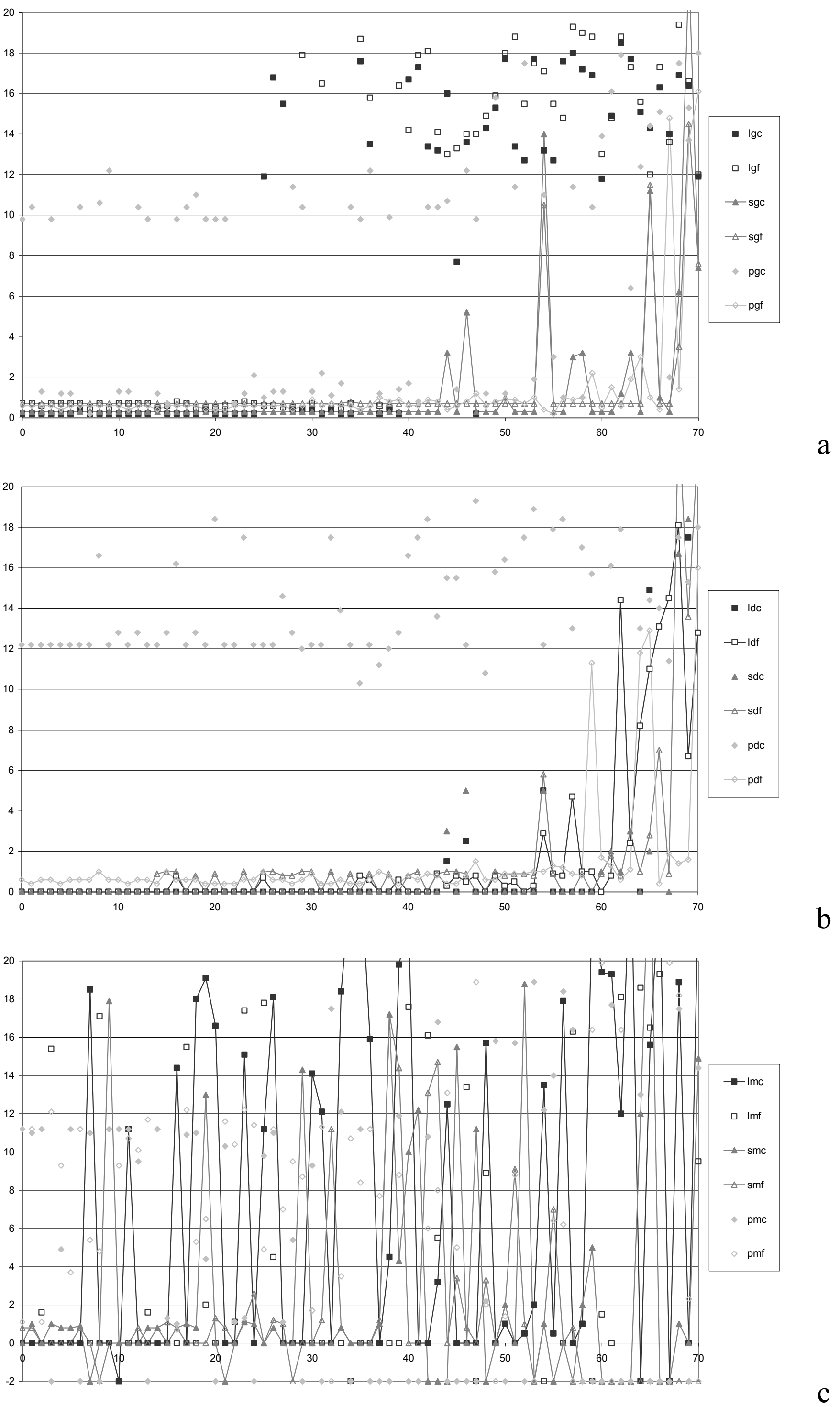


Figure 4. Error measures received in the tests of the compared methods. a: GHT; b: DAHT; c: MIHT. For MIHT the negative error means the algorithm did not converge after 100 iterations. Letters in the denotations of the series: first letter for the test – “l” for *large rectangle onto small*, “s” for *small onto larg*e, “p” for *phantom*; second letter for the method – “g” for GHT, “d” for DAHT, “m” for MIHT; third letter for version – “c” for crisp, “f” for fuzzy. In each subfigure, the graph for the test and version for which the result was the most robust was marked with points with a line, and the remaining graphs – only with points.

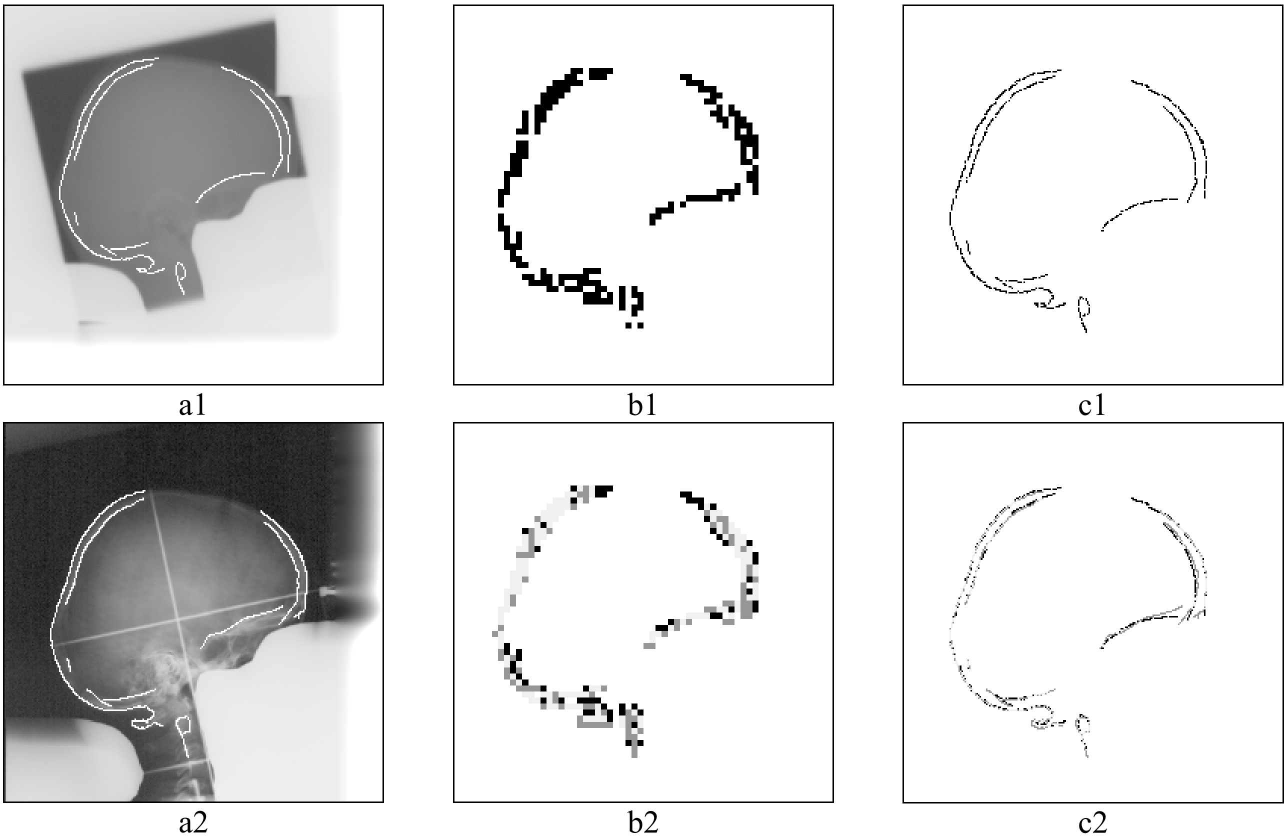


Figure 5. a1, a2: edges of anatomical structures – registered features – in images 250*250, portal (overlaid) and simulation (reference), respectively. b, c: portal images, optimally transformed to the simulation images, for resolution 1:4 (63*63, magnified) and 1:1 (250*250), respectively. b1, c1: transformed overlaid images. b2, c2: transformed overlaid images shown together with reference ones; bright grey – pixels having correspondence (134 and 363, respectively); dark grey – overlaid pixels with no correspondence (90 and 481, respectively); black – reference pixels with no correspondence (62 and 349, respectively).

4 CONCLUSION

Three methods derived form the concept of the Hough transform were compared in the application to the image registration problem. The version of the registration considered was the fully automatic, interpolating, linear registration of the images with features. The fact that the interpolating methods were considered is important from the point of view of the way in which the errors in data were treated: the methods were intended to dismiss the erroneous data (erroneous feature points) and to overlay the correct data (the truly corresponding feature points).

The compared methods were: the Generalised Hough Transform method, otherwise known as the generic method for finding, in the binary image, the pattern given with another binary image, the method of Direct Accumulation of Parameters (DAHT), known on the grounds of photogrammetry as the Geometrically Invariant Parameter Space Clustering (GIPSC), and the Modified Iterative Hough Transform (MIHT). The first two methods – GHT and DAHT – are computationally complex and require considerable memory, but the presently available computational equipment make them interesting in some selected ranges of application, due to their simplicity and generality. The considerable robustness against errors of their fuzzy versions, or in other words, the resistance to erroneous and missing data, is their important advantage. In this respect, the DAHT seems more robust then GHT. The GHT, DAHT and MIHT are free from any assumptions related to the shapes formed by the feature pixels.

In contrast to the two other methods, the MIHT is much quicker and requires little memory. However, is not always applicable, and its convergence for the given application should be tested prior to the final use. It does not exhibit the feature of robustness to errors in data, either.

The DAHT method is competitive to the GHT method for small numbers of feature points in the registered images, irrespectively of the resolution of the accumulator array, that is, irrespectively of the expected precision. It is then well suited for automatic estimation of the transformation parameters for the images in reduced resolution. The GHT is competitive for larger numbers of feature points, but it becomes less efficient for larger resolutions of the accumulator. It is easy to form the fuzzy versions of the both methods, which relaxes the precision-certainty tradeoff typical for the evidence accumulating methods derived from the Hough transform. There exists a possibility of introducing the constraints which would reduce the computational complexity of DAHT into the acceptable range, even for larger numbers of feature points.

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